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On the Construction of Life Tables, illustrated by a new Life Table of the Healthy Districts of England. By W. FARR, ESQ., M.D., F.R.S.*

THE *Transactions of the Royal Society* contain the first Life Table. It was constructed by Halley, who discovered its remarkable properties, and illustrated some of its applications. The Breslau observations did not supply Halley with the data to frame an accurate table, for reasons which will be immediately apparent; but the conception is full of ingenuity, and the form is one of the great inventions which adorn the annals of the Royal Society.

Tables have since been made correctly representing the vitality of certain classes of the population; and the form has been extended so as to facilitate the solution of various questions.

In deducing the English Life Tables from the national returns, I have had occasion to try various methods of construction; and I now propose to describe briefly the nature of the Life Table, to lay down a simple method of construction, to describe an extension of its form, and to illustrate this by a new table representing the vitality of the healthiest part of the population of England.

The Life Table is an instrument of investigation; it may be called a *biometer*, for it gives the exact measure of the duration of life under given circumstances. Such a table has to be constructed for each district and for each profession, to determine their degrees of salubrity. To multiply these constructions, then, it is necessary

* Extracted, with permission, from the *Philosophical Transactions*, 1860.

to lay down rules, which, while they involve a minimum amount of arithmetical labour, will yield results as correct as can be obtained in the present state of our observations.

I. GENERAL DESCRIPTION OF A LIFE TABLE. (*See* Table C.)*

A Life Table represents a *generation of men* passing through *time;* and time under this aspect, dating from birth, is called age. In the first column of a Life Table, *age* is expressed in *years,* commencing at 0 (birth), and proceeding to 100 or 110 years, the extreme limit of observed lifetime.

If we could trace a given number of children, say 100,000, from the date of birth, and write the numbers down that die in the first year, living therefore less than one year, against 0 in the Table, and on succeeding lines the numbers that die in the second, third, and every subsequent year of age until the whole generation had passed away, these numbers would form a *Table of Mortality,* showing at what ages 100,000 lives become extinct.

Again, if the 100,000 children were followed, and the numbers living on the first, on the second, and on every subsequent birthday until none was left, the column of numbers would constitute a *Table of Survivorship.* So if, of 100,000 children born at a given point of time, the numbers dying (d_x) in each subsequent year were written in one column, and the numbers surviving (l_x) at the end of each year in another column, the two primary columns of the Life Table would be formed.

It is evident that if one of these columns is known, the other may be immediately deduced from it; for if, of 100,000 children born, 10,295 die in the first year of age, 3,005 in the second year of age, it follows that the numbers living at the end of one year must be 89,705, at the end of two years 86,700. Upon adding the column (d_x) from the bottom up to the number against any age (x), the sum will represent the whole of the numbers *dying after that age;* and, consequently, the numbers *living at that age,* as shown in the collateral column (l_x).

The 100,000 children born at the same moment, and counted *annually* to determine the numbers *living* at *the end of every year,* would, by our Table, completely pass away in less than 107 years. If another generation of 100,000, born a year afterwards, were followed, the numbers dying in the various years of age would not be very different, the circumstances remaining the same; and the numbers of those entering each year of age would vary inconsider-

* The tables and plates referred to in this and the following pages will be found, with the concluding portion of the paper, in the next number.

ably from those of the first series. If 100,000 children again were born at annual intervals, and were subject to an invariable law of mortality, they would form a community of which the numbers living at each age would be represented by the successive numbers (l_x) in the Life Table. The sum of these numbers, by the new Table of Healthy Districts, would be 4,951,908. The births are here assumed to take place simultaneously, at annual intervals; immediately before the births, therefore, in such a community, its population would be 4,851,908, to which it would fall progressively from 4,951,908 by 100,000 successive deaths in the year. The average number constantly living would be some number between 4,951,908 and 4,851,908; and it would be very nearly the mean of these limiting numbers.

In the ordinary course of nature, the births in a community take place in remittent succession; and if it is assumed that the 100,000 births occur at equal intervals over every year, it is evident that, at any given date, a certain number will be found living at all the intermediate points of age between 0 to 1 year, 1 to 2, 2 to 3, and all the remaining years of age. The population in the above instance would be found, by enumeration, to be nearly 4,899,665.

The annual *births* would be 100,000 in such a community. The annual deaths would also be 100,000; and by taking out the deaths at each year of age, from the parish registers of a single year, the second column (d_x) of the Life Table would be found. By adding this column of deaths up, and entering the sum of the numbers year by year against every year of age (x), the third column (l_x) of the Life Table would be obtained; for it has been already shown that the numbers attaining any age x are equal to the numbers dying at that age and all the subsequent ages. From the registers of the deaths, a table of the numbers of the *population living* in a parish *so constituted* could be immediately determined without any enumeration. Its deviations from the truth would be accidental; and they would be set right by taking the mean of many years. So, also, from a simultaneous enumeration of the *numbers living in each year of age*, the two columns d_x and l_x of the life could be constructed without reference to any registry of the deaths at different ages.

The *mean age at death* in such a community would express the mean lifetime, or the expectation of life at birth; and the product of the number expressing the annual births multiplied into the mean age at death would give the numbers of the population.

The facts which a Life Table expresses in numbers may be

represented by the lines of a figure; age (x) being indicated by the abscissas measured from 0, the *numbers living* (l) at each age by the ordinates of a curve line, and the numbers living between any two ages by the plane surface within the two ordinates, the curve line, and the corresponding portion of the abscissa. The relative numbers living at the ages 20 and 21 are seen in the two lines of Plate XLII., fig. 1, over the ages 20 and 21; if the deaths in the intervening year all occurred immediately after the age 20 was attained, the numbers living would also be represented by the parallelogram having its two sides equal to the ordinate over 21, and for its base the portion of the abscissa between 20 and 21; but if all the deaths occurred only the instant before the age 21 was attained, the height of the parallelogram would be represented by the ordinate over the age of 20. The deaths occur at intervals between the two ages, so the numbers living, and the *lifetime* which is passed between the two ages, are correctly represented by the curvilinear area.

The deaths in each year of age are called the *decrements of life.* They are represented by the differences in the lengths of the successive ordinates. Thus, by cutting off a small portion of the ordinate at the age 20, the ordinate at the age 21 is obtained; this small portion, shown in Plate XLII., represents the decrement of life in that year of age. It will be observed that the decrements vary at every year of age; and this is more evident when they are exhibited on the larger scale of Plate XLII., fig. 2. The decrement in the first year is large; in the first five years the decrements of life are considerable; at the age of 10 to 15 they fall to their minimum, slowly increase to the age of 56, increase more rapidly until the maximum is attained at the age of 75, then decline gradually to 85, and, after that, more rapidly, until every life is extinct at the age 107 by this table.

II. PRINCIPLES OF CONSTRUCTION—THE FUNDAMENTAL COLUMN l_x.

The conditions of the hypothesis upon which the preceding reasoning rests are never precisely realised in nature—in the first place, the number of births fluctuates, increases, or decreases, from year to year; and the deaths fluctuate still more, rarely equalling the births in number. Immigration and emigration interfere. Under these circumstances, tables such as those which Halley, Price, and others, made from the observations on the *deaths alone* are never accurate, and require correction to give approximate

results. If it be assumed that the law of mortality remains invariable, and that migration does not interfere, then the nature of the correction to be applied to a table framed from the deaths alone will become immediately apparent by an example. The births increase in England. Let the annual births in a portion of the community be doubled in sixty years—thus, be 50,000 in 1796, and 100,000 in 1856; then the deaths of persons of the age of 60 in 1856 must be doubled to obtain the deaths which would have happened at that age if the annual births sixty years before these deaths had been 100,000. If the births have been accurately registered, formulæ for correcting the ordinary table drawn up from the deaths at different ages will be suggested by the above considerations.

I now proceed to describe another method which has been adopted in framing the Table C, and is applicable wherever (1) the number of annual births, (2) the numbers of the population living at definite periods of age, (3) the deaths at the corresponding ages during a certain number of years, in any community, are ascertained by observation. This method is not open to the previous objections.

The aim is to obtain equations which will describe the curve lines (Plate XLII., fig. 1) of the Life Table in the most direct way; and these equations may be deduced from the determined rate of mortality at certain intervals of age.

The relative numbers living at two ages, 20 and 21, can evidently be found from an equation which expresses the relation of the average numbers living and dying between those ages during a given time. This can be determined very nearly; for, although the ages of the living are not ascertained with exact precision at the census, still, by taking all the numbers living at the ages 15, 16, 17 years, up to 24 and under 25, together, the aggregate represents very nearly the numbers living in that decenniad of life. The deaths at the same ages are obtained with at least equal accuracy from the registers of deaths. By this process, and by extending the observations over five or more years, a number of facts is obtained sufficiently great to yield average results; and it may be assumed that the ratio of the living at the ages 15–25* to the dying in a year at the same ages, 15–25, represents the annual rate of mortality at the exact age 20. So also the mortality rate at the ages 30, 40, 50, and other ages, may be determined. As observations grow more exact, and the facts are multiplied, the

* By this, 15 and *under* 25 years of age is understood, and so in all similar cases.

intervals of age may be diminished to 5 years, and ultimately to 1 year.

In determining the *rate* of *mortality*, a given number of persons living a year is considered equivalent to twice that number living half a year, or to half the number living two years.

Thus, if nd represent the deaths in n years out of a number amounting, on an average, to P during the same years, then $\frac{nd}{n\text{P}}=m=$ the rate of mortality, or the proportions of death in a *year* (always taken as the unit of time) out of *one year* of *lifetime*. It is found, from all the observations hitherto made on a large scale, that the rate of mortality varies at every interval of age; but at the same age it may, for the present purpose, be considered invariable under similar circumstances.

m_x therefore varies in every moment of age; but I have employed it to express the mean annual rate of mortality during the year following the year of age x, $\therefore \frac{d_x}{\text{P}_x}=m_x$, where d_x indicates the deaths, P_x the year of lifetime, after the year of age x. The m_x is the expression of the force of the causes that induce death, of the death-force, *vis mortalis*; and its reciprocal $\frac{1}{m_x}=u_x$ measures the forces that sustain life, the *vis vitalis*.

The vital force, under natural circumstances, may, by one hypothesis, be sufficient to sustain a whole generation alive for seventy or eighty years, and then suddenly collapse. The Life Table, if this hypothesis were true, would be represented by the *parallelogram* in which the curve of the Life Table is inscribed (Plate XLII., fig. 1).

By the hypothesis of Demoivre,* the rate of mortality is such, that, at the age of 20, 1 in 66 living at the beginning dies before the end of the year, leaving 65, 64, 63, 62, 61, to enter on each year of age until, at the age of 86, all are dead.

Upon this hypothesis the relative numbers living up to the age 86 form an arithmetical progression; and the deaths in the equal times are equal out of the diminishing numbers living. The rate of mortality increases on this hypothesis, as age advances, in the same ratio as $n-\frac{1}{2}:1$; where n is the difference between the actual age x and 86. It is called the complement of life. The Life Table upon this hypothesis has equal decrements, and might be represented on Plate XLII., fig. 1, by drawing a diagonal line

* See *Treatise of Annuities on Lives;* Preface to 2nd edition.

through the parallelogram. Its deviation from the true curve on this scale is evident; but it is also evident that a series of straight lines, which would nearly represent the true curve, may be drawn from point to point of all the ordinates.

If the causes of death act with equal intensity at all ages, they may be represented by any simple external cause destroying an equal *proportion* of the numbers living in equal intervals of time. Thus, if 1,600 men were distributed equally over ground where they were exposed to certain dangers, represented by successive discharges of musketry, which, at every discharge, shot down one-half of the numbers remaining, they would be reduced successively from 1,600 to 800, to 400, to 200, to 100, to 50, and so on *ad infinitum*, if a fraction of a living man could be conceived; the numbers living at each year of age in a Life Table would not decrease at *these rates*, but they *would decrease* at a constant rate if the dangers at every stage of life remained *constant* and equally *great*. The numbers of the living at successive ages would be in geometrical progression, and would be represented by the ordinates of the logarithmic curve.

The law of mortality can only be derived from observation, and it is found to be less simple than either of these hypotheses implies. It can, however, be represented nearly by equations at different periods of age. Upon inspecting Table A, it will be seen that, at the age 55–65, which may be represented by the exact age 60, the mortality is such, that 2,162 women die in a year out of a number equal to 100,000 living a year; and the mortality, which is the ratio of the dying to the living in a unit of time, here set down as a year, is, therefore, $m=\cdot02162$. Again: the mortality at the age of 70 is ·04992, at the age of 80 it is ·11866, and at the age of 90 it is ·26711. The mortality increases rapidly, and is more than doubled every ten years. The four numbers differ little from the terms of a geometrical progression, the logarithms of which have a constant difference. Let the rate at which the mortality increases be r, and $r^{10}=2\cdot3116$, and the first term (m) be ·02177; then a series of numbers will be formed differing little from those which express the value of m at decennial intervals of age.

Values of m at the precise age x.—*Females.*

Age (x).	60.	70.	80.	90.
By observation .	·02162	·04992	·11866	·26711
By hypothesis .	·02177	·05033	·11633	·26891

Note.—It may be assumed that m at 60 is the mean value of m in its range from $m_{59\frac{1}{2}}$ to $m_{60\frac{1}{2}}$; and so in other cases.

The *annual rate* of the increase of m from the age of 55 to 95 is $r=1{\cdot}0874$; and if m is the mortality at any age after 55, then $m_z=mr^z=$the mortality at z years after the age at which m is taken. The common logarithm of r is $=\lambda r={\cdot}03639$.

The mortality (m) of males at corresponding ages is higher than the mortality of females; but the rate of increase as age advances is nearly the same.

The value of m for females at the age of 20 is ·00765, and the mortality increases at the rate of nearly one-seventh part every ten years. The exact value of r is 1·0149, and $\lambda r={\cdot}006423$.

Values of *m*.—*Females.*

Age.	20.	30.	40.	50.
By observation .	·00765	·00894	·00998	·01192
By hypothesis .	·00760	·00882	·01022	·01185

By these observations, in the healthy districts the mortality (m) of men at the ages 15 to 45 is lower than the mortality of women at the same ages; yet, during that period, the rate of increase r is nearly the same for the two sexes. From the age of 40 to 50, and 50 to 60, the mortality of males increases at a rate intermediate between the rates of manhood and mature age.

Limits of Ages.		
	Females.	
15 to 55 or 20 to 50	$r=1{\cdot}0149$	$\lambda r={\cdot}00642$
55 to 95 or 60 to 90	$r=1{\cdot}0874$	$\lambda r={\cdot}03639$
	Males.	
15 to 45 or 20 to 40	$r=1{\cdot}0148$	$\lambda r={\cdot}00640$
55 to 95 or 60 to 90	$r=1{\cdot}0874$	$\lambda r={\cdot}03640$

The subjoined Table exhibits the series of values for m derived from the hypothesis of two constant rates, and from direct observation. The values of r for females may be evidently applied to males in every period, except in the ten years of age, 40 to 50.

Mortality (*m*) *of males and females,* (1) *derived from observation, and* (2) *from the hypothesis that* m *increases at the preceding rates.*

Precise Age.	ANNUAL MORTALITY TO 100 CONSTANTLY LIVING AT EACH AGE (m).			
	Males.		Females.	
	By observation.	By hypothesis.	By observation.	By hypothesis.
20	·691	·696	·765	·760
30	·818	·807	·894	·882
40	·928	·935	·998	1·022
50	1·273	1·083	1·192	1·185
60	2·294	2·329	2·162	2·177
70	5·486	5·385	4·992	5·033
80	12·817	12·451	11·866	11·633
90	28·350	28·785	26·711	26·891
100	40·000?	66·550?	45·000?	62·160?

The observations on the numbers living and dying of the age of 95 and upwards are exceedingly uncertain; and it is probable that many of the persons believed to be 100, &c., are really persons five or ten years younger; so that these values of m_x, by the hypothetical method, are probably as correct as the direct numbers.

I shall now notice briefly the application of this hypothesis, first suggested by Mr. Gompertz, and applied by him to the interpolation of the Northampton and other Tables.* Mr. Edmonds, in 1832, extended the "Theory," and applied it to the construction of three Life Tables.† He gave an elegant formula, similar in principle to that of Mr. Gompertz, from which the curve of a Life Table can be deduced, upon the above hypothesis.

In the equation $\frac{s}{t}=v$, where s indicates space, t time, v velocity, the units of measure must be fixed before numbers can be inserted in the general expression; and then v will express, in the measure that has been applied to space, the number of such units of space described in *one* unit of time. Here v is a ratio—it is the rate at which the body moves; and, in the same manner, m, in the equation $\frac{d}{l}=m$, is the *rate of dying*—that is, as I shall express it, the *mortality;* or it is the ratio of the dying to the living in a given unit of time, the time during which the deaths occur being of precisely the same duration as the time during which the living are under observation—

l (living during 1 year) : d (dying during a year) :: 1 (year of life) : m.

If for l the number 100,000 is substituted, it is assumed that immediately a death occurs another life is substituted; and as the time is a year, then 760 will represent the value of d at the age 20, according to the preceding Table; $\therefore m=\cdot 00760$. If the *time,* instead of *one year,* be the *thousandth part* of one year, then $m=\cdot 0000076$; and if the time be infinitely short, m will be infinitely small: m is a ratio; the quantity of life existing during the time is represented by 1, and the quantity of life destroyed by a fraction, m. Whether the life inheres in the first organic molecule after conception, in the infant, or in the man, the vital action has a certain force of continuance, which is constantly varying; and the amount of this *force* that is *extinguished* at a given instant of time will be represented by the force of mortality—namely, by

* *Philosophical Transactions*, 1825; paper by B. Gompertz, Esq., F.R.S.

† *Life Tables founded on the Discovery of a Numerical Law regulating the Existence of every Human Being, &c.* By T. R. Edmonds, B.A., 1832.

m at that instant. Then let the age $x=z+a$, where a represents the number of years up to the age at which a given rate (r) of increase of m begins, then $z=x-a$; and the mortality at any instant of age, in an instant of time at the end of z years or parts of years, will be mr^z. Now, let y represent the living at that precise age, then the decrement of y in an infinitely short time will be $-dy=ymr^z dx$; the dy being negative, as it is taken in a direction opposite to that in which the ordinate y of the curve is assumed to be drawn. Transferring y to the other side of the equation, this becomes $-\frac{dy}{y}=mr^z dz$; and integrating both sides, we have ($\lambda_\epsilon y$ being put for the hyperbolic logarithm of y, and $\lambda_\epsilon c$ for the difference between the constants of the two integrals)—

$$\lambda_\epsilon c-\lambda_\epsilon y=\lambda_\epsilon \frac{c}{y}=\frac{mr^z}{\lambda_\epsilon r}; \quad \dots\dots \quad (1)$$

$$\therefore \ \lambda_\epsilon y=y_\epsilon c-\frac{mr^z}{\lambda_\epsilon r}, \quad \dots\dots\dots \quad (2)$$

and

$$\lambda_\epsilon c=\lambda_\epsilon y+\frac{mr^z}{\lambda_\epsilon r}. \quad \dots\dots\dots \quad (3)$$

When z is made zero, let $y=1$; then $\lambda_\epsilon y$ will also disappear, and $\lambda_\epsilon c=\frac{m}{\lambda_\epsilon r}$. Upon substituting this value of $\lambda_\epsilon c$ in equation (2), it becomes—

$$\lambda_\epsilon y=\frac{m}{\lambda_\epsilon r}-\frac{mr^z}{\lambda_\epsilon r}=\frac{m}{\lambda_\epsilon r}(1-r^z). \quad \dots \quad (4)$$

Upon passing to the numbers, equation (4) becomes

$$y=\epsilon^{\frac{m}{\lambda_\epsilon r}(1-r^z)}$$

$=$the value of y (taken as 1 at the origin) at the end of z years.

Let λ denote the common logarithm with the base 10, then $\lambda_\epsilon y=\frac{\lambda y}{k}$, where k is the modulus of the common system of logarithms; as also

$$\lambda_\epsilon c=\frac{km}{\lambda r}, \quad \text{and} \quad \frac{mr^z}{\lambda_\epsilon r}=\frac{kmr^z}{\lambda r}.$$

Equation (2) becomes, after the required substitutions,

$$\frac{\lambda y}{k}=\frac{km}{\lambda r}-\frac{kmr^z}{\lambda r}$$

and

$$\lambda y=\frac{k^2 m}{\lambda r}(1-r^z); \quad \dots\dots \quad (5)$$

so the equation becomes finally

$$y=10^{\frac{k^2m}{\lambda r}(1-r^z)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This is the form given by Mr. Edmonds, and is convenient for use.

By making z successively 1, 2, 3, ... up to any number less than the number of years of age within which r remains constant, the number l_x being known, the number living at any other age within that range will be obtained by multiplying l_x by the corresponding value of y. Thus, if y_{10} is the value of y when $z=10$ in equation (6), then, putting l_{20} for the numbers living at the age 20, the living at the age 30 will be $y_{10}\times l_{20}=l_{30}$.

This hypothesis does not express the facts deduced from the observations exactly. If m_z could be expressed exactly over more than 20 years by $m_z=m_0r^z$, the first differences (δ^1) of the logarithms in the series following would, in a certain number of cases, be equal.

Females in Healthy Districts of England.

Precise Age.	Annual Rate of Mortality.	Logarithms of the Annual Mortality.	First Decennial Differences of λm_x.	Second Decennial Differences of λm_x.
x.	m*.	λm.	δ^1.	δ^2.
20	·00765	$\bar{3}$·8835	·0677	−·0197
30	·00894	$\bar{3}$·9512	·0480	·0290
40	·00998	$\bar{3}$·9992	·0770	·1817
50	·01192	$\bar{2}$·0762	·2587	·1047
60	·02162	$\bar{2}$·3349	·3634	·0126
70	·04992	$\bar{2}$·6983	·3760	−·0236
80	·11866	$\bar{1}$·0743	·3524	−·1259
90	·26711	$\bar{1}$·4267	·2265	
100	·45000	$\bar{1}$·6532		

The inequalities in the second differences vary in every separate class of observations, but there is generally a tendency in the first and in the second differences to increase, over a certain extent of the series. The error of the hypothesis is slight if the rate of increase (r), of which λ·00677 is the logarithm in the case in hand, is only assumed to remain uniform for the ten years 20 to 30, or for the one year 20 to 21. Now, let the number living at the age

* Here, at the age 20, m is the mean mortality that rules over the age $19\frac{1}{2}$ to $20\frac{1}{2}$ years of exact time.

20 be represented by l_{20}, and the number living at the age 21 by l_{21}; then put $\frac{l_{21}}{l_{20}}=p_{20}$. Here it is evident that if l_{20} and p_{20} be known, l_{21} is determined immediately by the equation $l_{21}=l_{20}\times p_{20}$. But p_{20} is the value of y in the equation $y_1=10^{\frac{k^2m}{\lambda r}(1-r^z)}$, when z is put $=1$. Taking the numbers from Table A, we have $m=\cdot00765$ at the precise age $20=(19\frac{1}{2}+20\frac{1}{2})\frac{1}{2}$; and $\lambda m=\bar{3}\cdot8835130$, $\lambda r=\cdot0067728$, and $\therefore r=1\cdot015717$; k is put for the modulus of the common logarithms, $\therefore \lambda k^2=\bar{1}\cdot2755686$; $k(\lambda r)$ is the complement of the *logarithm* of (λr).

λk^2	$\bar{1}\cdot2755686$
λm	$\bar{3}\cdot8835130$
$k(\lambda r)$	$2\cdot1692317$
$\lambda(1-r)$	$\bar{2}\cdot1963697$
$-\cdot0033472$	$\bar{3}\cdot5246830$
$\bar{1}\cdot9966528$	

As the factor $(1-r)$ is negative, it makes the exponent of 10 negative; and, upon taking the complement of this, the logarithm of y is found to be $\bar{1}\cdot9966528$. This is also the logarithm of $p_{20}=\cdot99232$, and it enables us to pass, in the construction of a Life Table, from the living at the age of 20 to the living at 21. If we obtain the several values p_x at every year of age, the whole of the Life Table can be constructed.

It will be found that p_x is always a fraction, and it does not differ very much from $1-m_x$; but while m_x* shows the *deaths* in a year out of a *unit* of *life* (which may consist of any *number* of individual *lives* constantly kept up), p_x shows how much out of a *unit of the same life* at the beginning of a year, the dead not being replaced, *survives a year* after the age x; and $1-p_x$ is the amount of loss which occurs in the same year out of a unit of life at its commencement. Thus, as $p_{20}=\cdot99232$, it follows that $1-p_{20}=\cdot00768$. In the same year of age, 20 to 21, the mortality is $m_{20}=\cdot00771$, or $\cdot00003$ more than $(1-p_{20})$. If the unit of life is made 100,000 living at the age 20, then 99232 will survive, and 768 will die in the ensuing year of age. But if it is assumed that the deaths take place at equal intervals, it may also be assumed that the number of lives (100,000) being constantly sustained, the accession of 768 new lives takes place at equal intervals, conse-

* m serves to indicate the mean mortality in the year following the exact age x.

quently that they are under observation half a year on an average, giving the equivalent of $\frac{768}{2}=384$ years of lifetime at the age 20 to 21. Now, out of this number (384), at that age *three* die when the mortality is m_{20}. This accounts for the difference of ·00768 and ·00771, the former occurring in a year out of a unit of life of which the waste is not replaced.

From these considerations, it may be inferred that, if m_x is known, p_x may be deduced from it, upon the hypothesis of equal decrements through the year, by the formula $p_x=\frac{1-\frac{1}{2}m_x}{1+\frac{1}{2}m_x}=\frac{2-m_x}{2+m_x}$. Thus, m_{20} being ·0077072, we have $\frac{\cdot 9961464}{1\cdot 0038536}=\cdot 99232$,* as before. The λp_{20}, by the previous method, is $\bar{1}\cdot 9966528$, and by this method it is the same. By either of the methods the value of p_x may be deduced for the subsequent ages, and p_{20}, p_{30}, p_{40} . . . . . p_{90}, p_{100} will be obtained. These values are here given, and it will be seen that the results by the two methods are nearly identical at all ages, except the last two, when the observations themselves become less exact.

Females.

Age (x).	$\lambda p_x=\lambda y_1=10^{\frac{\kappa^2 m}{\lambda r}(1-r)}$.	$\lambda p_x=\lambda\left(\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}\right)$.
20	$\bar{1}$·9966528	$\bar{1}$·9966527
30	·9960967	·9960967
40	·9956263	·9956264
50	·9946669	·9946676
60	·9902049	·9902073
70	·9773538	·9773557
80	·9463182	·9462643
90	·8809176	·8801776

It will be observed that the fraction $p=\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$ approximates to $1-m$ as m becomes less; for, upon developing it into a series, $p=1-m+\frac{1}{2}m^2-\frac{1}{4}m^3+\frac{1}{8}m^4$ . . . . And taking m infinitely small, the terms after the first two may be neglected.

* m at the precise age 20 is nearly ·00765. The increase in this mortality from the age 20 to $20\frac{1}{2}$, the middle of the year of age 20 to 21, is obtained by adding $\frac{1}{2}\lambda r$, as above given, to $\lambda m_{19\frac{1}{2}}$, that is, to the log. of $(m_{19\frac{1}{2}}+m_{20\frac{1}{2}})\frac{1}{2}$; $\therefore m_{20}=\cdot 0077072$

$$\begin{array}{rl} \lambda m_{19\frac{1}{2}} & \bar{3}\cdot 8835130 \\ \frac{1}{2}\lambda r & 0\cdot 0033864 \\ \hline \lambda m_{20} & \bar{3}\cdot 8868994 \end{array}$$

The values of $m_0, m_1 \ldots\ldots m_5$ may be obtained by the method already described; but it rarely happens that the population living at each year of age is accurately enumerated at the census; and, besides inaccuracies of statement, the numbers living at each of the early years of age fluctuate considerably, so that the numbers of children living of each year of age in 1851 do not represent the average numbers living of those ages in the five years 1849 to 1853 for instance.

The following method is less exceptionable. It may be assumed *for this purpose*, (1) that the births registered in the year 1848 represent the births in that year; (2) that the births are equally distributed over the years in which they occur; and, consequently, (3) that the *mean date* of all *the births* in the two years 1848, 1849, was immediately before January 1, 1849. The *half* of the births in those two years will consequently represent pretty accurately the number of births out of which the deaths of children *under one year* of age happened in the year 1849; and the deaths and survivors can be followed by this method year by year, as is evident in the annexed scheme:—

Age.		
0	$\frac{1}{2}$ (births 1848, 1849)=mean annual births of which the mean date is January 1, 1849.	
	minus deaths under age 1 in 1849	
1		=surviving on January 1, 1850.
	minus deaths age (1 to 2) in 1850	
2		=surviving on January 1, 1851.
	minus deaths age (2 to 3) in 1851	
3		=surviving on January 1, 1852.
	minus deaths age (3 to 4) in 1852	
4		=surviving on January 1, 1853.
	minus deaths age (4 to 5) in 1853	
5		=surviving on January 1, 1854.

By commencing with the mean number of births in the years 1849, 1850, and deducting the deaths, a similar series may be obtained; and thus a succession of similar series may be deduced, the mean of which will supply the ordinary series $l_0, l_1, l_2, l_3, l_4, l_5$ of a Life Table.

These series are liable to various disturbances. If all the births are not registered, the *rate* of mortality is overstated; if all the deaths are not registered, or if the children are carried off as emigrants, the decrements of life are understated. The annual number of births fluctuates, and now increases, in England; they are in excess also in the early months of the year. Several of the disturbances are slight, and some of them are in opposite directions.

The results can also be, and have been, checked by the results of the other method. The values of m_7 and m_{12} are deduced by dividing the annual deaths at the ages 5 to 10 and 10 to 15 by the mean population at those ages. The interpolation of the series λp_x from λp_3 to λp_{20} succeeds, taking λp_3, λp_7, λp_{12}, and λp_{20}, as the fixed points of the series, and λp_{12} being adjusted to allow for the turn of the curve.

The Tables A, B, and C supply the data from which the Life Table of Healthy English Districts was deduced. One or two arithmetical examples of the application of the method adopted in the earlier ages are also supplied.

III. INTERPOLATION.

We have therefore determined the values of λp_x at certain ages. The values of λp_x at the intervening ages may be determined by changing the value of r and making z successively 1, 2 . . . . 10 in the formula (p. 131). They may also be interpolated for every year of age by the method of finite differences; and, upon the whole, this method is preferable to any other. The logarithms of p_x are required, and to them it will be convenient to apply the interpolation directly. Any number of differences beyond four becomes cumbersome, and it will be therefore sufficient to give the general formula, which can be employed in deriving the first of either four or three orders of differences.

Investigation of Formulæ—Intervals equal.

Let any numbers of a series be so related that u_n, the nth from the first, u_0, is determined by the equation (1)—

$$u_n = u_0 \begin{matrix}+\\-\end{matrix} \frac{n}{1}\delta^1 \begin{matrix}+\\+\end{matrix} \frac{n(n-1)}{1.2}\delta^2 \begin{matrix}+\\-\end{matrix} \frac{n(n-1)(n-2)}{1.2.3}\delta^3 \begin{matrix}+\\+\end{matrix} \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\delta^4. \quad . \quad (1)$$

δ^1, δ^2, δ^3, and δ^4,* the first differences of the four orders, are unknown; they can all be determined from any five values of u_n. Now, let n be successively $1x$, $2x$, $3x$, $4x$; then the coefficients of u_0, u_{1x}, u_{2x}, u_{3x}, u_{4x}, can be found to give the values of δ^1, δ^2, δ^3, and δ^4, in four equations. But when x is ten or more, the coefficients become large and the numerical calculation laborious; it is, therefore, well to obtain the numerical values of δ^4, δ^3, δ^2, δ^1, in succession. Thus, if the series is ascending or descending, the following are convenient forms—the upper rows of signs are used in the *ascending*, the lower rows in the *descending*, series:—

* It will be borne in mind that these imply first differences, or $\delta^1 u_0$, $\delta^2 u_0$, $\delta^3 u_0$, $\delta^4 u_0$.

$$\delta^4 = \frac{\begin{smallmatrix}+\\+\end{smallmatrix} u_{4x} \begin{smallmatrix}-\\-\end{smallmatrix} 4u_{3x} \begin{smallmatrix}+\\+\end{smallmatrix} 6u_{2x} \begin{smallmatrix}-\\-\end{smallmatrix} 4u_x \begin{smallmatrix}+\\+\end{smallmatrix} u_0}{x^4} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (2)$$

$$\delta^3 = \frac{\begin{smallmatrix}+\\-\end{smallmatrix} u_{3x} \begin{smallmatrix}-\\+\end{smallmatrix} 3u_{2x} \begin{smallmatrix}+\\-\end{smallmatrix} 3u_x \begin{smallmatrix}-\\+\end{smallmatrix} u_0}{x^3} \begin{smallmatrix}-\\+\end{smallmatrix} \tfrac{3}{2}(x-1)\delta^4. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (3)$$

$$\delta^2 = \frac{\begin{smallmatrix}+\\+\end{smallmatrix} u_{2x} \begin{smallmatrix}-\\-\end{smallmatrix} 2u_x \begin{smallmatrix}+\\+\end{smallmatrix} u_0}{x^2} \begin{smallmatrix}-\\+\end{smallmatrix} (x-1)\delta^3 \begin{smallmatrix}-\\-\end{smallmatrix} \frac{(7x^2-18x+11)}{12}\delta^4. \quad . \; . \; . \; . \; . \qquad (4)$$

$$\delta^1 = \frac{\begin{smallmatrix}+\\-\end{smallmatrix} u_x \begin{smallmatrix}-\\+\end{smallmatrix} u_0}{x} \begin{smallmatrix}-\\+\end{smallmatrix} \frac{x-1}{2}\delta^2 \begin{smallmatrix}-\\-\end{smallmatrix} \frac{(x^2-3x+2)}{6}\delta^3 \begin{smallmatrix}-\\+\end{smallmatrix} \frac{(x^3-6x^2+11x-6)}{24}\delta^4. \quad . \qquad (5)$$

It is necessary to be careful in deducing the successive values of δ from the values preceding; and, before commencing their use, their accuracy should be tested by inserting them in the checking equation—

$$u_{4x} = u_0 \begin{smallmatrix}+\\-\end{smallmatrix} \frac{4x}{1}\delta^1 \begin{smallmatrix}+\\+\end{smallmatrix} \frac{4x(4x-1)}{1.2}\delta^2 \begin{smallmatrix}+\\-\end{smallmatrix} \frac{4x(4x-1)(4x-2)}{1.2.3}\delta^3$$
$$\begin{smallmatrix}+\\+\end{smallmatrix} \frac{4x(4x-1)(4x-2)(4x-3)}{1.2.3.4}\delta^4. \quad . \; . \; . \; . \; . \qquad (6)$$

x may be any number. If only four terms are given, δ^3 is assumed to be constant; and δ^4 being 0, all the terms into which it enters disappear. The above formulæ, if this is borne in mind, are applicable when δ^4, δ^3, or δ^2, are assumed to be constant, and serve, therefore, to supply the differences, when there are one, two, three, or four orders, by the most expeditious method.

In constructing the Life Table, x was made 10 from the age of 20, and on inserting the numbers, the equations (2, 3, 4, 5, 6) became

$$\delta^4 = \frac{\begin{smallmatrix}+\\+\end{smallmatrix} u_{40} \begin{smallmatrix}-\\-\end{smallmatrix} 4u_{30} \begin{smallmatrix}+\\+\end{smallmatrix} 6u_{20} \begin{smallmatrix}-\\-\end{smallmatrix} 4u_{10} \begin{smallmatrix}+\\+\end{smallmatrix} u_0}{10{,}000} \quad . \; . \; . \; . \; . \; . \; . \qquad (7)$$

$$\delta^3 = \frac{\begin{smallmatrix}+\\-\end{smallmatrix} u_{30} \begin{smallmatrix}-\\+\end{smallmatrix} 3u_{20} \begin{smallmatrix}+\\-\end{smallmatrix} 3u_{10} \begin{smallmatrix}-\\+\end{smallmatrix} u_0}{1{,}000} \begin{smallmatrix}-\\+\end{smallmatrix} 13\tfrac{1}{2}\delta^4. \quad . \; . \; . \; . \; . \; . \; . \qquad (8)$$

$$\delta^2 = \frac{\begin{smallmatrix}+\\+\end{smallmatrix} u_{20} \begin{smallmatrix}-\\-\end{smallmatrix} 2u_{10} \begin{smallmatrix}+\\+\end{smallmatrix} u_0}{100} \begin{smallmatrix}-\\+\end{smallmatrix} 9\delta^3 \begin{smallmatrix}-\\-\end{smallmatrix} 44\tfrac{1}{4}\delta^4. \quad . \; . \; . \; . \; . \; . \; . \qquad (9)$$

$$\delta^1 = \frac{\begin{smallmatrix}+\\-\end{smallmatrix} u_{10} \begin{smallmatrix}-\\+\end{smallmatrix} u_0}{10} \begin{smallmatrix}-\\+\end{smallmatrix} 4\tfrac{1}{2}\delta^2 \begin{smallmatrix}-\\-\end{smallmatrix} 12\delta^3 \begin{smallmatrix}-\\+\end{smallmatrix} 21\delta^4. \quad . \; . \; . \; . \; . \; . \; . \qquad (10)$$

The checking equation is—

$$u_{40} = \mathop{+}_{+} u_0 \mathop{+}_{-} 40\delta^1 \mathop{+}_{+} 780\delta^2 \mathop{+}_{-} 9880\delta^3 \mathop{+}_{+} 91390\delta^4. \quad . \quad . \quad (11)$$

If three orders of differences are used, the checking equation is—

$$u_{30} = \mathop{+}_{+} u_0 \mathop{+}_{-} 30\delta^1 \mathop{+}_{+} 435\delta^2 \mathop{+}_{-} 4060\delta^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

After adding or subtracting any constant to or from a series of numbers, the differences remain the same; and if consecutive terms are multiplied or divided by the same factor, the differences are multiplied or divided by that factor. Thus $(b+a)-(c+a)=b-c$, and $ab-ac=a(b-c)$. Advantage is taken of these properties to reduce any one of the terms in the equations to *zero.*

Thus, let the logarithms to be interpolated be the following—values of p_{20}, p_{30}, p_{40}, and p_{50}, taken from the column headed *males*, Table B, then they may, among other ways, be interpolated as follows:—

As $\bar{1}\cdot9969724$ is the contracted expression of $(\cdot9969724-1)$, we have—

Age.			
20	$\bar{1}\cdot9969724=-\cdot0030276$	(1) Multiplying each term by 10,000,000—that is, striking out the decimal point and the two adjoining ciphers—and (2) then subtracting from each 30,276, the values of $u_x=\lambda p_x$ to be operated on become	$u^0=-00000$
30	$\bar{1}\cdot9964260=-\cdot0035740$		$u_{10}=-\ 5464$
40	$\bar{1}\cdot9959051=-\cdot0040949$		$u_{20}=-10673$
50	$\bar{1}\cdot9943048=-\cdot0056952$		$u_{30}=-26676$

By inserting these values with their negative signs in the equations, and taking the upper signs, the three differences are found—that is,

$$\delta^3=-11\cdot049; \quad \delta^2=101\cdot991; \quad \text{and } \delta^1=-872\cdot7715.$$

The differences are now divided by 10,000,000—that is, ciphers are added to their left-hand side, so that the above decimal point may be moved seven places in that direction—and the operation may be thus commenced. By adding the differences successively to each other and to $\lambda p_{20}=\bar{1}\cdot9969724$, the successive values are found of λp_{21}, λp_{22}, λp_{23} . . . . λp_{50}, up to and including λp_{58}, for males, where the series joins naturally the subsequent series, commencing at λp_{59}.

δ^3.	δ^2.	δ^1.	λp_x.
$-$·000,0011,0490	·000,0101,9910	$-$·000,0872,7715	$\bar{1}$·996,9724,0000
(constant)	·000,0090,9420	$-$·000,0770,7805	$\bar{1}$·996,8951,2285
		$-$·000,0679,8385	$\bar{1}$·996,8180,4480
			$\bar{1}$·996,7500,6095

In the actual operation the δ^3 is *subtracted* from δ^2, δ^2 from δ^1, and δ^1 from λp_x; it is, therefore, convenient to substitute for their present values the complements of δ^3 and δ^1, as thus all the series become additive.

As $\lambda l_{20} + \lambda p_{20} = \lambda l_{21}$, and $\lambda l_{21} + \lambda p_{21} = \lambda l_{22}$, and, generally, $\lambda l_x + \lambda p_x = \lambda l_{x+1}$, it is evident that the λp_x is the *first difference* of the series λl_x; and the whole series, λl_x, from λl_{20} to λl_{58}, may be formed as in the subjoined example, where δ^3 becomes δ^4, δ^2 becomes δ^3, and so on.

Healthy Districts—Males.

δ^4 (constant)

9·999,9988,9510

Age.	δ^3.	δ^2.	$\delta^1 = \lambda p_x$.	$u_x = \lambda l_x$.
20	0·000,0101,9910	9·999,9127,2285	9·996,9724,0000	4·584,1951,2769
21	0·000,0090,9420	9·999,9229,2195	9·996,8851,2285	4·581,1675,2769
22	0·000,0079,8930	9·999,9320,1615	9·996,8080,4480	4·578,0526,5054
23			9·996,7400,6095	4·574,8606,9534
24				4·571,6007,5629

Note.—The four last figures in the decimal portion of the series λp_x and in λl_x may, in practice, be omitted.

The corresponding values of λp_x, in the column headed "Females," Table B, are interpolated in the same way; and the λp_{60}, λp_{70}, λp_{80}, and λp_{90}, are interpolated by the same methods, the series being continued backwards to λp_{57} and forwards to λp_{105}; the actual observations of age after the age of 90 furnishing results less liable than those thus obtained, which bring a generation of 100,000 to their last end in 107 years. The successive values of λp_x, in the period from the age of 3 to the age of 19 inclusive, are derived from λp_3, λp_7, λp_{12}, and λp_{20}, which represent u_0, u_4, u_9, and u_{17}. As the terms of the series are here at unequal distances, the first differences cannot be derived from the preceding formulæ. The δ can in this and similar cases be derived from the proper equations by substituting figures for letters. But three literal equations supply formulæ for finding the three first differences from any four terms of series of the kind which have been dis-

cussed; u_0, which has a troublesome coefficient, can always be reduced to *zero*, and is, therefore, omitted. The first given term being u_0, let the second, u_x, be the xth from u_0, and u_y be the yth, u_z the zth from u_0; here $x<y<z$; then the following equations give the differences*:—

$$\delta^3=\frac{6\left\{(y-x)\frac{u_z}{z}-(z-x)\frac{u_y}{y}+(z-y)\frac{u_x}{x}\right\}}{(y-x)\{(z-1)(z-2)-(y-1)(y-2)\}-(z-y)\{(y-1)(y-2)-(x-1)(x-2)\}}. \quad (13)$$

$$\delta^2=\frac{2}{y-x}\left\{\frac{u_y}{y}-\frac{u_x}{x}-\{(y-1)(y-2)-(x-1)(x-2)\}\frac{\delta^3}{6}\right\} \quad \ldots\ldots\ldots \quad (14)$$

$$\delta^1=\frac{u_x}{x}-(x-1)\frac{\delta^2}{2}-(x-1)(x-2)\frac{\delta^3}{6} \quad \ldots\ldots\ldots\ldots \quad (15)$$

By making $y=2x$, and $z=3x$, these equations assume the same forms as equations (3), (4), (5), with the term δ^4 struck out.

Putting $x=4$, $y=9$, and $z=17$, the three preceding equations become those which were actually used in constructing the series p_3 to p_{19}: u_0 is reduced to zero and is not used.

$$\delta^3=\frac{45u_{17}-221u_9+306u_4}{13260} \quad \ldots\ldots\ldots \quad (16)$$

$$\delta^2=\frac{4u_9-9u_4-300\delta^3}{90} \quad \ldots\ldots\ldots \quad (17)$$

$$\delta^1=\frac{u_4-6\delta^2-4\delta^3}{4} \quad \ldots\ldots\ldots \quad (18)$$

Checking equation.

$$u_{17}=u_0+17\delta^1+136\delta^2+680\delta^3 \quad \ldots\ldots\ldots \quad (19)$$

* *A useful Table in applying the above formulæ.*

x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.
20	342	34	1056	47	2070
21	380	35	1122	48	2162
22	420	36	1190	49	2256
23	462	37	1260	50	2352
24	506	38	1332	51	2450
25	552	39	1406	52	2550
26	600	40	1482	53	2652
27	650	41	1560	54	2756
28	702	42	1640	55	2862
29	756	43	1722	56	2970
30	812	44	1806	57	3080
31	870	45	1892	58	3192
32	930	46	1980	59	3306
33	992				

Table of first differences in the Life Table of Healthy Districts of England.

Age x.	λl_x.	$\lambda p_x = \delta^1$.	δ^2.	δ^3.	δ^4.
			MALES.		
3	4·631,5849,0000	9·993,2422,0000	0·001,2416,1260,934	9·999,8012,4393,666	0·000,0141,9648,567
20	4·584,1951,2769	9·996,9724,0000	9·999,9127,2285	0·000,0101,9910	9·999,9988,9510
59	4·403,7768,0454	9·990,6137,0980	9·998,9756,9020	9·999,9704,0800	9·999,9843,4520
60	4·394,3905,1434	9·989,5894,0000	9·998,9460,9820	9·999,9547,5320	9·999,9843,4520
	Note.—The last series p_x was carried backwards from λp_{60} to λp_{59}.				
			FEMALES.		
3	4·623,2586,0000	9·993,2928,0000	0·001,2164,1598,794	9·999,7874,2556,561	0·000,0170,4566,365
20	4·570,6868,3846	9·996,6528,0000	9·999,9241,5455	0·000,0060,2930	9·999,9994,2530
57	4·405,2189,6826	9·992,9332,3725	9·999,0836,2675	0·000,0123,2100	9·999,9838,1950
60	4·381,2818,8126	9·990,2049,0000	9·999,0720,4825	9·999,9637,7950	9·999,9838,1950
	Note.—The last series p_x was carried backwards from λp_{60} to λp_{57}.				

(20)

A series of the form $v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2}$ is required in rendering the Life Table applicable to the solution of questions in annuities and life insurance.

The logarithms of the series are obtained by making the first term of the new series $\lambda(v^x l_x)$, and the first term of the first order of differences $\lambda(vp_x) = \lambda v + \lambda p_x = \delta^1$; the δ^2, δ^3, and δ^4 of the original series remaining unchanged. Taking the interest of money at 3 per cent., $v = \frac{1}{1{\cdot}03}$; and $\lambda v = \bar{1}{\cdot}9871627{,}753$.

The derivation of the new series from this value of λv, and from the above Table (males), is shown in the annexed example. Any value of v^x may be introduced in the same way.

$\delta^4 = 9{\cdot}9999988{,}951$

Age.	δ^3.	δ^2.	$\lambda(vp_x) = \delta^1$.	$u_0 = \lambda(l_x v^x)$.
20	0·0000101,991	9·9999127,2285	9·9841351,7530	4·3274506
	·0000090,942	·9999229,2195	·9840478,9815	·3115858
		·9999320,1615	·9839708,2010	·2956337
			·9839028,3625	·2796045
				·2635074

In describing the first English Life Table, I ventured to express the belief that the chances of life may ultimately be calculated by Mr. Babbage's machine.* Mr. Babbage's conception has been realized in the original and ingeniously-constructed machine of the Messrs. Scheutz, which was favourably reported upon by a com-

* Letter to the Registrar-General, in Appendix (p. 352) to his *Fifth Annual Report*, year 1843.

mittee of the Royal Society. The first differences to be inserted in the machine can be immediately deduced from those given above; and we may hope ere long to see the logarithms of Life Tables, for single and for joint lives, printed from types cast in moulds stamped by the machine now in the course of construction by the Messrs. Donkin, for Her Majesty's Government, at the instance of the Registrar-General.

(*To be continued.*)

On the Clearing of the London Bankers. By SIR JOHN W. LUBBOCK, BART., F.R.S., *formerly Treasurer of the Royal Society, and Vice-Chancellor of the University of London.**

Atque equidem, extremo nî jam sub fine laborum
Vela traham, et terris festinem advertere proram;
Forsitan et pingues hortos quæ cura colendi
Ornaret, canerem, biferíque rosaria Pæsti:
Quóque modo potis gauderent intyba rivis;
Et virides apio ripæ, tortúsque per herbam
Cresceret in ventrem cucumis: nec sera comantem
Narcissum, aut flexi tacuissem vimen acanthi,
Pallentésque hederas, et amantes litora myrtos.

THE operation of the clearing has the effect of enabling all the payments from one bank to another to be performed without the passing of bank-notes; and the result is, that if any bank has to receive from the clearing, say, £50,000, the account of that bank at the Bank of England is better by £50,000 at 9 o'clock the next morning; and if a bank has to pay into the clearing £50,000, the amount of that bank is worse by £50,000.

The following description of the mode of conducting the clearing is taken from Mr. Babbage's *Treatise on the Economy of Machinery and Manufactures* (second edition, page 124):—

"In London all checks paid in to bankers pass through what is technically called '*The Clearing House.*' In a large room in Lombard-street, about thirty clerks from the several London bankers take their stations, in alphabetical order, at desks placed round the room; each having a small open box by his side, and the name of the firm to which he belongs in large characters on the wall above his head. From time to time, other clerks from every house enter the room, and, passing along, drop into the box the checks due by that firm to the house from which this distributor is sent. The clerk at the table enters the amount of the several checks in a book previously prepared, under the name of each bank to which it is due.

* The object of this paper is one in the attainment of which the readers of this journal are probably little interested; but the paper itself affords so remarkable an instance of the application of the doctrine of probabilities to the ordinary affairs of life, that we have not hesitated to insert it.—ED. *A. M.*